SENIOR HIGH MATHEMATICS

MATH 13

COURSE OF STUDIES

IMPLEMENTATION - SEPTEMBER 1989

CURRICULUM DESIGN BRANCH

Alberta
EDUCATION

A. PROGRAM RATIONALE AND PHILOSOPHY

In order for students to be able to set goals and make informed choices, they need an array of thinking and problem-solving skills. Fundamental to this is an understanding of mathematical techniques and processes that will enable them to apply the basic skills necessary to address everyday mathematical situations, as well as acquire higher order skills in logical analysis and methods for making valid inferences.

A knowledge of mathematics is essential for a well-educated citizenry. However, the need for and use of mathematics in the life of the average citizen is changing. Emphasis has shifted from the memorization of mathematical formulae and algorithms toward a more dynamic view of mathematics as a precise language, used to reason, interpret and explore. There continues to be a need for the logical development of concepts and skills as a basis for the appropriate use of mathematical information to solve problems. Moreover, the use of available technology along with techniques such as estimation and simulation, incorporated with more traditional problem-solving techniques, are the tools by which mathematical problems are solved.

Change in the way in which mathematics is used is necessitating a concurrent change in the emphases of mathematics education. Students need an expanded list of fundamental concepts but will also need to understand the ideas that make up those concepts and how they are related. They also require a familiarity with their applications. Most important, students have to be able to solve problems using the mathematical processes developed, and be confident in their ability to apply known mathematical skills and concepts in the acquisition of new mathematical knowledge. In addition, the ability of technology to provide quick and accurate computation and manipulation, to enhance conceptual understanding and to facilitate higher order thinking, should be recognized and used by students.

The majority of students who enter high school exhibit mainly concrete operational behaviours with regard to mathematics. It is recognized that high school mathematics courses include many abstract understandings which students are expected to acquire. The course content of the high school mathematics program is cognitively appropriate for the students and should be presented in a way which is consistent with the students' ability to understand.

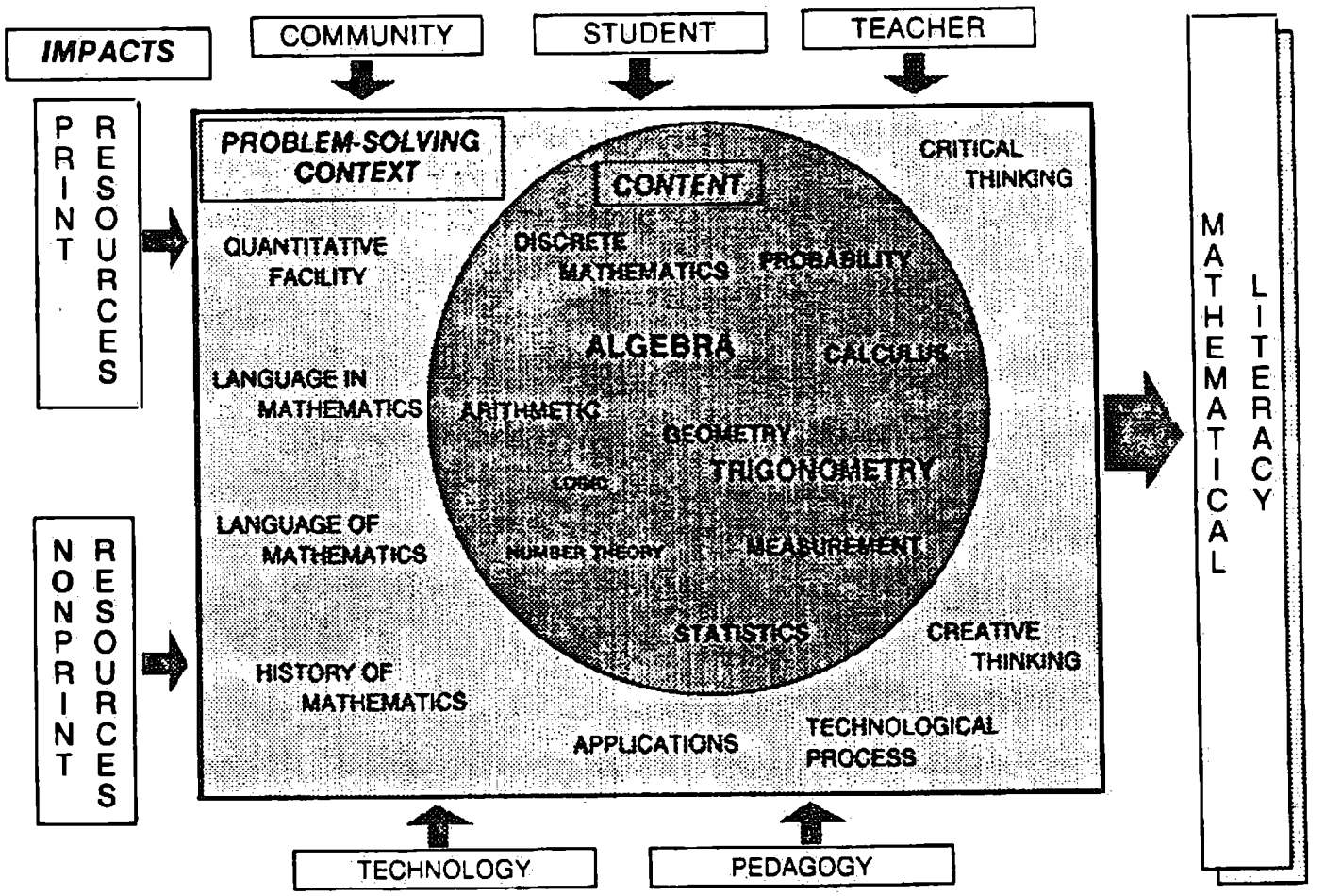
The Senior High School Mathematics Program, by providing course sequences commensurate with differing abilities, interests and aspirations, is designed to enable students to have success in mathematics. As well, it reflects the changing needs of society, and provides students with the mathematical concepts, skills and attitudes necessary to cope with the challenges of the future.

B. GENERAL LEARNER EXPECTATIONS

Upon conclusion of their senior high mathematics education, students will be expected to be mathematically literate. More specifically, *students will be expected to*

- have achieved understanding of the basic mathematical concepts, and developed the skills and attitudes needed to become responsible and contributing members of society
- apply basic mathematical skills and concepts in practical situations
- have developed the skills, concepts and attitudes that will ensure success in the mathematical situations that occur in future educational endeavours, employment and everyday life
- have developed the skills, concepts and attitudes that will enable the acquisition of mathematical knowledge beyond the conclusion of secondary education
- have developed critical and creative thinking skills
- be able to communicate mathematical ideas effectively
- understand how mathematics can be used to investigate, interpret and make decisions in human affairs
- understand how mathematics can be used in the analysis of natural phenomena
- understand and appreciate the positive contributions of mathematics, as a science and as an art, to civilization and culture.

A Model for Mathematical Literacy



A Model for **Mathematical Literacy** outlines the factors that effect what and how students learn as they become mathematically literate. The model is fluid in that the content can be learned within a problem-solving context which engages any of a number of vehicles as the learning focus. At the same time, the students involved in the learning situation are affected in what and how they learn by forces which have impact upon them.

The **Content** of the high school mathematics program is that body of knowledge that is to be acquired by students. In the various high school mathematics courses, it is made up of topics that can be categorized into one or more of the listed strands.

Within each course in the high school mathematics program, students will focus on problem solving. The **Problem-Solving Context** refers to the instructional emphases within which the specific content expectations can be acquired. The various entries indicated within the model suggest processes that belong to the problem-solving context and may be used by students as vehicles for learning the content.

The **Impacts** on the Problem-Solving Context are those skills, attitudes and experiences that are possessed by the students and teachers involved, as well as the resources they may use throughout the learning process. They include the influence exerted by the culture and beliefs of the community as reflected by the school. The effectiveness of the context in enabling a student to acquire the content is dependent upon the skilful management by the teacher of those items that impact upon a student's learning.

Mathematical literacy refers to the ability to use mathematical skills, concepts and processes to address successfully the mathematical situations which one encounters.

C. SPECIFIC LEARNER EXPECTATIONS

The specific learner expectations are what a student is expected to accomplish in a particular course. For mathematics these include attitudes, concepts and skills, as well as expectations about problem solving and the use of technology.

Attitudes are general dispositions toward mathematics education and social issues. Attitude expectations are listed at the beginning of each course and stated as "Students will be encouraged to develop...".

A concept is an abstract or general idea about specific instances that have common properties or an identifiable relationship to one another. Each unit is broken into a number of concepts which are the major ideas to be addressed. Concepts may be presented as mathematical definitions or theorems, or as statements of mathematical ideas or abstractions. In the course of study, concepts are numbered with a single digit and stated as "Students will be expected to demonstrate an understanding that...".

Supporting each concept are a number of skills. Skills are intellectual or physical capabilities that will be developed within the context of the particular concept. In the program of studies, skills are numbered with two digits separated by a period, (n.m) and stated as "Students will be expected to...".

In addition, related to particular concepts and skills, are problem-solving expectations. These expectations are of two general types. Some identify specific types of problems which must be solved while others identify problem-solving behaviours or techniques to be learned by students. Problem-solving expectations occur throughout the specific learner expectations. They suggest that students should have the background skills and knowledge necessary to achieve the expectations successfully, using problem-solving techniques. In the program of studies, problem-solving expectations are numbered with three digits separated by periods (n.m.l).

Technology expectations identify specific areas in which calculator and/or computer technology will be applied by students as tools to be used for calculation, manipulation or graphing, or to aid in the analysis of problems. In the program of studies, technology expectations are numbered with three digits separated by periods (n.m.l).

Throughout the learner expectations, the words verify and prove appear. For the purposes of the Senior High Mathematics Program, they are interpreted as:

- Verify - to substantiate the validity of an operation, solution, formula or theorem through the use of examples which may or may not be generalized;
- Prove - to substantiate the validity of an operation, solution, formula or theorem in general and to provide logical arguments for each step in the process.

The major part of the content of each high school mathematics course consists of topics required of all students who take the course. The required content comprises 80% of the course and contains the concepts, skills and attitudes that all students are expected to acquire. As well, the required portion of all courses includes specific expectations in regard to problem solving and the use of technology.

Each course will include a compulsory component comprising 20% of the course, made up of elective material which will be consistent with the content and expectations of the required component. The purpose of the elective is to provide for enrichment, remediation, or innovative or experimental presentations or activities. It is not intended to provide acceleration or advanced placement. However, horizontal enrichment and extension is appropriate and students should have access to elective material that serves their individual needs and interests.

Through the use of problems and applications, students will be able to see the need for certain concepts and mathematical procedures, to understand concepts and acquire facility with procedures, and to apply and review concepts and procedures already learned. The emphasis on problem solving in high school mathematics has shifted from finding answers to routine "word problems", to the application of mathematical concepts, skills and processes to develop and discover mathematics within a wide variety of problem situations.

Problem solving has been integrated into all of the courses in the Senior High Mathematics Program and is explicitly mentioned throughout the learner expectations. In addition, specific expectations for problem solving are included at the beginning of each course. It is not intended that problem solving be presented as units of study; however, it is expected that upon completion of the course, students will have acquired all of the listed skills and concepts.

Students will be expected to learn how and when to use a calculator and have a demonstrated proficiency in estimation and mental arithmetic. To use calculators effectively, they must be able to judge the reasonableness of an answer and understand the importance of making a judgment about the result of a calculation.

Throughout the learner expectations, specific applications of calculators and computers are identified. These define explicitly the areas in which students are expected to use technology in their study of mathematics. Many of these expectations indicate that a particular technology be used in an investigation or for analysis. It is in these situations that the use of technology enables students to engage in critical and creative thinking and problem solving.

Evaluation of students in the Senior High Mathematics Program will involve assessment of the level of achievement of all of the learner expectations, including concepts, skills and attitudes, as well as problem solving and technology expectations. For more information regarding evaluation consult the Teacher Resource Manual for Senior High Mathematics.

Attitudes

Within the study of the content of this course, it is expected that *students will be encouraged to*

- be confident in their mathematical knowledge and in their ability to acquire new knowledge;
- demonstrate persistence, resolve, flexibility and ingenuity in finding the solution to problems;
- develop an attitude of intellectual curiosity and openness to new ideas, insights and change in the pursuit of mathematical knowledge;
- exhibit an attitude of curiosity and spontaneity and appreciate creativity and innovation in representing situations mathematically;
- value and be aware of the importance of communication skills in mathematics;
- appreciate and develop a positive attitude toward the usefulness of computational competence, mathematical processes and problem solving skills which are used in the decision making and modeling processes in our society;
- value the contributions of mathematics to our culture and civilization.

Problem Solving

Problem solving is to be integrated into all areas of the high school mathematics program because a major purpose of studying mathematics is to learn to solve problems. A student who has successfully completed this course will understand that thinking and problem-solving skills are required in order to explain the world, and that mathematics can provide some of the tools for problem solution.

Students will understand that problems can be solved in many ways, and that problems can be routine, requiring only the application of a known procedure or algoarithm, or non-routine, requiring the development of a process or the conducting of an investigation to solve them.

Students will be expected to use a variety of procedures to help them understand mathematical problems. In particular, they will be expected to

- read the problem thoroughly
- identify and clarify key components
- restate the problem using familiar terms
- evaluate the given information as to whether it is insufficient or extraneous
- interpret pictures, charts and graphs
- determine any hidden assumptions
- ask relevant questions
- identify given, needed and wanted information
- diagram or model the problem situation
- use suitable notation
- determine valid inferences.

Students will be expected to develop a variety of strategies for use in the solution of mathematical problems. In particular, they will be expected to

- conduct an investigation
- use estimation and approximation
- develop equations or use formulae
- use flow charts
- make lists and charts
- look for patterns
- work backward
- break the problem into smaller parts
- look for a simpler or related problem
- make diagrams or models
- use manipulatives
- choose and sequence a series of mathematical operations
- sketch the graph of a problem situation
- establish procedures to gather and organize data.

Students will be expected to develop a variety of skills which can be used to carry out the plan for the solution of a problem. In particular, they will be expected to

- apply selected strategies.
- present ideas clearly
- document the solution process
- utilize appropriate group behaviours
- use calculators and computers
- evaluate problem-solving strategies for effectiveness
- alter or abandon non-productive strategies
- search for additional information
- ask questions
- be open to inspirations, intuitions and "bright ideas".

Students will be expected to employ a variety of skills to help them look back over the solution of a problem. In particular, they will be expected to

- determine the reasonableness of an answer
- explain the solution in oral or written form
- consider the possibility of additional solutions
- search for other strategies and processes of solution
- create and solve similar problems
- note the characteristics that will be identifiable in similar problems
- make a generalization.

Number Skills

1. *Students will be expected to* demonstrate an understanding of the four arithmetic operations on rational numbers, carried out in conjunction with the operations of rounding and estimation.

   Students will be expected to

   1.1 perform calculations on rational numbers, using mental calculation, calculators and paper-and-pencil techniques, including multiple operations involving the rules for order of operations

      1.1.1 use calculators in calculations involving rational numbers
      1.1.2 solve problems involving calculations with rational numbers

   1.2 perform calculations using percents

      1.2.1 use calculators in calculations that involve the use of percents
      1.2.2 solve problems that involve the use of percent calculations

   1.3 calculate numerical squares and square roots

      1.3.1 use electronic calculators to find numerical squares and square roots.

2. *Students will be expected to* demonstrate an understanding of the laws and properties pertaining to calculations involving expressions with exponents.

   Students will be expected to

   2.1 perform calculations involving powers with integral bases and integral exponents

      2.1.1 verify the properties of zero and negative exponents through the use of numeric examples
      2.1.2 use calculators to evaluate powers with integral bases and integral exponents.

Polynomials and Factoring

1. *Students will be expected to* demonstrate an understanding that polynomials can be used to describe some mathematical situations in which values vary.

Students will be expected to

1.1 identify the following as they occur in the study of polynomials: term, variable, factor, monomial, binomial, trinomial, polynomial, numerical coefficient, degree, exponent, base, power
1.2 evaluate a polynomial for given values of the variables
1.3 use polynomials to describe mathematical situations.

2. *Students will be expected to* demonstrate an understanding of the laws and properties pertaining to calculations involving expressions with exponents.

Students will be expected to

2.1 use the multiplication law of exponents for powers with literal bases and whole number exponents, $a^m \times a^n = a^{m+n}$

2.1.1 verify the multiplication law of exponents through the use of examples

2.2 use the power law for exponents, $(a^m)^n = a^{mn}$

2.2.1 verify the power law for exponents through the use of examples

2.3 use the division law of exponents for powers with literal bases and whole number exponents, $\frac{a^m}{a^n} = a^{m-n}$

2.3.1 verify the division laws for exponents through the use of examples

2.4 use zero and negative exponents

2.4.1 verify the laws for zero and negative exponents through the use of examples.

3. *Students will be expected to* demonstrate an understanding that calculations and operations on polynomials depend on the application of the laws of operations for numbers and powers.

Students will be expected to

3.1 add and subtract polynomials
3.2 multiply polynomials by monomials
3.3 multiply binomials by binomials
3.4 recognize the expansion of the square of a binomial and the product of conjugate binomials as special cases
3.5 divide polynomials by monomials.

4. *Students will be expected to* demonstrate an understanding that polynomials can be written as a product of their factors.

Students will be expected to

4.1 factor polynomials that have a monomial common factor
4.2 factor trinomials of the form $ax^2 + bx + c$, where $a = 1$ or is a common factor
4.3 recognize and factor perfect square trinomials recognizing them as a special case
4.4 recognize and factor difference of squares polynomials, recognizing them as a special case
4.5 factor polynomials that require a combination of methods in order to be fully factored

4.5.1 solve problems that involve factoring of polynomials.

Coordinate Geometry and Graphing

1. *Students will be expected to* demonstrate an understanding that points and lines can be represented on a Cartesian plane and have characteristics that can be defined and measured.

   Students will be expected to

   1.1 use the formula for the slope of a line passing through two given points

      1.1.1 solve problems that involve the use of the slope formula
      1.1.2 verify the relationships between the slopes of parallel and perpendicular lines by diagraming pairs of lines on a Cartesian plane
      1.1.3 solve problems that involve the relationships between the slopes of parallel and perpendicular lines

   1.2 identify the characteristics of lines that have positive, negative, zero and undefined slopes
   1.3 use the Pythagorean relation to find the distance between two points on the Cartesian plane

      1.3.1 solve problems that involve finding the distance between two points on the Cartesian plane

   1.4 find the midpoint of a segment

      1.4.1 solve problems that involve finding the midpoint of a line segment.

2. *Students will be expected to* demonstrate an understanding that the ordered pairs which satisfy linear equations correspond to coordinates on the Cartesian plane, and can be used to graph the equation on the Cartesian plane.

   Students will be expected to

   2.1 find sets of ordered pairs that satisfy a linear equation, and use them to sketch the graph of the equation
   2.2 find the x- and y-intercepts of the graphs of linear equations
   2.3 graph linear equations by drawing a graph through the x- and y-intercepts
   2.4 write linear equations in the slope-intercept form, $y = mx + b$
   2.5 graph linear equations using the slope and y-intercept

      2.5.1 investigate the effects of the parameters m and b in $y = mx + b$
      2.5.2 use calculators or computers to draw the graphs of any linear equations

   2.6 recognize and graph the equations of lines that are parallel to the x- and y-axes.

Relations

1. *Students will be expected to* demonstrate an understanding that phenomena can be related and that these relations can be represented by graphs and tables of values.

   Students will be expected to

   1.1 sketch graphs that describe physical phenomena or everyday occurrences including the identification of the dependent and independent variables, and appropriately labeled axes

1.1.1 solve problems involving physical phenomena and everyday occurences by modelling them graphically

1.2 read and interpret graphs drawn to describe the relationships between real life variables
1.3 interpolate and extrapolate to find information from given graphs and tables of data.

2. *Students will be expected to* demonstrate an understanding that some pairs of variables relate to each other directly or inversely, according to a relationship that can be written as an equation.

Students will be expected to

2.1 recognize relations in which the variables vary directly

2.1.1 solve direct variation problems
2.1.2 solve partial variation problems

2.2 recognize relations in which the variables vary inversely

2.2.1 solve inverse variation problems.

Statistics

1. *Students will be expected to* demonstrate an understanding that populations have certain characteristics, which can be determined by analyzing the characteristics of a suitable sample of the population.

Students will be expected to

1.1 determine a sample that will adequately and accurately represent a population
1.2 evaluate samples for bias, appropriateness of sample type, amount of data and randomness
1.3 draw inferences about the population from which the sample was taken and express it using the language of probability

1.3.1 determine a valid sample, design and carry out a simple survey, and make a valid inference about the population based on the results
1.3.2 defend or oppose a generalization that is made about a population from a sample.

2. *Students will be expected to* demonstrate an understanding that data collected from a sample must be organized, presented and analyzed in order that valid inferences can be made.

Students will be expected to

2.1 identify a situation, formulate an hypothesis, and plan what data needs to be collected
2.2 organize data using stem-and-leaf plots and box plots
2.3 choose and calculate appropriate measures of central tendency for sets of data
2.4 represent data using appropriate graphs
2.5 do simple visual analyses on sets of data organized in stem-and-leaf plots, box plots and graphs

2.5.1 draw and defend conclusions based on the visual analyses of sets of data

2.6 evaluate intuitively an analysis of data for the confidence with which the results can be accepted.

Geometry

1. *Students will be expected to* demonstrate an understanding that angles can relate to each other, in ways that can be determined by identifying particular characteristics of the segments that form them.

   Students will be expected to

   1.1 use the following terms: parallel, perpendicular, transversal, corresponding angles, alternate angles, interior angles
   1.2 use the relationships between vertically opposite angles to calculate the measures of angles

      1.2.1 verify that vertically opposite angles are equal

   1.3 use the relationships amongst angles made by parallel lines and a transversal

      1.3.1 verify the relationships amongst angles made by cutting parallel lines with a transversal.

2. *Students will be expected to* demonstrate an understanding that triangles can be uniquely determined by appropriate combinations of sides and angles.

   Students will be expected to

   2.1 construct a triangle, given two sides and the included angle (SAS), three sides (SSS) and two angles and a side (ASA)

      2.1.1 verify that any of the three conditions SAS, SSS or ASA uniquely determine a triangle.

3. *Students will be expected to* be able to demonstrate an understanding that similar triangles have the same shape but may differ in size, that angles are equal and that the ratios between corresponding sides are equal, and that congruent triangles are similar triangles of the same size .

   Students will be expected to

   3.1 use the relationship between the lengths of the sides of similar triangles to calculate the lengths of unknown sides

      3.1.1 verify that the ratios between corresponding sides of similar triangles are equal
      3.1.2 solve problems by using the relationships between the sides of similar triangles

   3.2 identify the postulate by which a pair of triangles can be shown to be congruent.

D. BASIC LEARNING RESOURCES

Carli, E.G. and K.E. Newton. Mathematics for a Modern World - Book 2. (Third Edition, Revised.) Agincourt, Ontario: Gage Educational Publishing Ltd., 1989.

ISBN 0-7715-3621-6

Ebos, Frank and Paul Zolis. Math Matters Book 2. (Second Edition.) Scarborough, Ontario: Nelson Canada, 1989.

ISBN 0-17-603082-4